CHAPTER 5
Risk and Rates of Return

- Stand-alone risk
- Portfolio risk
- Risk & return: CAPM / SML

Investment returns

The rate of return on an investment can be calculated as follows:

$$\text{Return} = \frac{(\text{Amount received} - \text{Amount invested})}{\text{Amount invested}}$$

For example, if $1,000 is invested and $1,100 is returned after one year, the rate of return for this investment is:

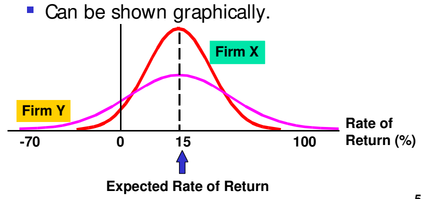
($1,100 - $1,000) / $1,000 = 10%.

What is investment risk?

- Two types of investment risk
  - Stand-alone risk
  - Portfolio risk
- Investment risk is related to the probability of earning a low or negative actual return.
- The greater the chance of lower than expected or negative returns, the riskier the investment.

Probability distributions

- A listing of all possible outcomes, and the probability of each occurrence.
- Can be shown graphically.



How do the returns of HT and Coll. behave in relation to the market?

- HT – Moves with the economy, and has a positive correlation. This is typical.
- Coll. – Is countercyclical with the economy, and has a negative correlation. This is unusual.

Return: Calculating the expected return for each alternative

$\hat{k}$ = expected rate of return

$$\hat{k} = \sum_{i=1}^{n} k_i P_i$$

$$\hat{k}_{HT} = (-22.\%)\,(0.1) + (-2\%)\,(0.2) + (20\%)\,(0.4) + (35\%)\,(0.2) + (50\%)\,(0.1) = 17.4\%$$

Summary of expected returns for all alternatives

	Exp return
HT	17.4%
Market	15.0%
USR	13.8%
T-bill	8.0%
Coll.	1.7%

HT has the highest expected return, and appears to be the best investment alternative, but is it really? Have we failed to account for risk?

Risk: Calculating the standard deviation for each alternative

σ = Standard deviation

$$\sigma = \sqrt{\text{Variance}} = \sqrt{\sigma^2}$$

$$\sigma = \sqrt{\sum_{i=1}^{n} (k_i - \hat{k})^2 P_i}$$

Standard deviation calculation

$$\sigma = \sqrt{\sum_{i=1}^{n} (k_i - \hat{k})^2 P_i}$$

$$\sigma_{T-bills} = \begin{bmatrix} (8.0 - 8.0)^2(0.1) + (8.0 - 8.0)^2(0.2) \\ + (8.0 - 8.0)^2(0.4) + (8.0 - 8.0)^2(0.2) \\ + (8.0 - 8.0)^2(0.1) \end{bmatrix}^{1/2}$$

$\sigma_{T-bills} = 0.0\%$ $\quad\sigma_{Coll} = 13.4\%$

$\sigma_{HT} = 20.0\%$ $\quad\sigma_{USR} = 18.8\%$

$\sigma_M = 15.3\%$

Comments on standard deviation as a measure of risk

- Standard deviation (σ_i) measures total, or stand-alone, risk.
- The larger σ_i is, the lower the probability that actual returns will be closer to expected returns.
- Larger σ_i is associated with a wider probability distribution of returns.
- Difficult to compare standard deviations, because return has not been accounted for.

Comparing risk and return

Security	Expected return	Risk, σ
T-bills	8.0%	0.0%
HT	17.4%	20.0%
Coll*	1.7%	13.4%
USR*	13.8%	18.8%
Market	15.0%	15.3%

* Seem out of place.

Coefficient of Variation (CV)

A standardized measure of dispersion about the expected value, that shows the risk per unit of return.

$$CV = \frac{\text{Std dev}}{\text{Mean}} = \frac{\sigma}{\hat{k}}$$

Risk rankings, by coefficient of variation

	CV
T-bill	0.000
HT	1.149
Coll.	7.882
USR	1.362
Market	1.020

- Collections has the highest degree of risk per unit of return.
- HT, despite having the highest standard deviation of returns, has a relatively average CV.

Investor attitude towards risk

- Risk aversion – assumes investors dislike risk and require higher rates of return to encourage them to hold riskier securities.
- Risk premium – the difference between the return on a risky asset and less risky asset, which serves as compensation for investors to hold riskier securities.

Portfolio construction: Risk and return

Assume a two-stock portfolio is created with $50,000 invested in both HT and Collections.

- Expected return of a portfolio is a weighted average of each of the component assets of the portfolio.
- Standard deviation is a little more tricky and requires that a new probability distribution for the portfolio returns be devised.

Calculating portfolio expected return

$\hat{k}_p$ is a weighted average:

$$\hat{k}_p = \sum_{i=1}^{n} w_i \hat{k}_i$$

$$\hat{k}_p = 0.5\,(17.4\%) + 0.5\,(1.7\%) = 9.6\%$$

An alternative method for determining portfolio expected return

Economy	Prob.	HT	Coll	**Port.**
Recession	0.1	-22.0%	28.0%	**3.0%**
Below avg	0.2	-2.0%	14.7%	**6.4%**
Average	0.4	20.0%	0.0%	**10.0%**
Above avg	0.2	35.0%	-10.0%	**12.5%**
Boom	0.1	50.0%	-20.0%	**15.0%**

$$\hat{k}_p = 0.10\,(3.0\%) + 0.20\,(6.4\%) + 0.40\,(10.0\%) + 0.20\,(12.5\%) + 0.10\,(15.0\%) = 9.6\%$$

Calculating portfolio standard deviation and CV

$$\sigma_p = \begin{bmatrix} 0.10\,(3.0 - 9.6)^2 \\ + 0.20\,(6.4 - 9.6)^2 \\ + 0.40\,(10.0 - 9.6)^2 \\ + 0.20\,(12.5 - 9.6)^2 \\ + 0.10\,(15.0 - 9.6)^2 \end{bmatrix}^{1/2} = 3.3\%$$

$$CV_p = \frac{3.3\%}{9.6\%} = 0.34$$

Comments on portfolio risk measures

- σ_p = 3.3% is much lower than the σ_i of either stock (σ_{HT} = 20.0%; $\sigma_{Coll.}$ = 13.4%).
- σ_p = 3.3% is lower than the weighted average of HT and Coll.'s σ (16.7%).
- ∴ Portfolio provides average return of component stocks, but lower than average risk.
- Why? Negative correlation between stocks.

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General comments about risk

- Most stocks are positively correlated with the market ($\rho_{k,m} \approx 0.65$).
- $\sigma \approx 35\%$ for an average stock.
- Combining stocks in a portfolio generally lowers risk.

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Creating a portfolio: Beginning with one stock and adding randomly selected stocks to portfolio

- σ_p decreases as stocks added, because they would not be perfectly correlated with the existing portfolio.
- Expected return of the portfolio would remain relatively constant.
- Eventually the diversification benefits of adding more stocks dissipates (after about 10 stocks), and for large stock portfolios, σ_p tends to converge to ≈ 20%.

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Breaking down sources of risk

Stand-alone risk = Market risk + Firm-specific risk

- Market risk – portion of a security's stand-alone risk that cannot be eliminated through diversification. Measured by beta.
- Firm-specific risk – portion of a security's stand-alone risk that can be eliminated through proper diversification.

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Failure to diversify

- If an investor chooses to hold a one-stock portfolio (exposed to more risk than a diversified investor), would the investor be compensated for the risk they bear?
  - NO!
  - Stand-alone risk is not important to a well-diversified investor.
  - Rational, risk-averse investors are concerned with σ_p, which is based upon market risk.
  - There can be only one price (the market return) for a given security.
  - No compensation should be earned for holding unnecessary, diversifiable risk.

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Capital Asset Pricing Model (CAPM)

- Model based upon concept that a stock's required rate of return is equal to the risk-free rate of return plus a risk premium that reflects the riskiness of the stock after diversification.
- Primary conclusion: The relevant riskiness of a stock is its contribution to the riskiness of a well-diversified portfolio.

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Beta

- Measures a stock's market risk, and shows a stock's volatility relative to the market.
- Indicates how risky a stock is if the stock is held in a well-diversified portfolio.

Calculating betas

- Run a regression of past returns of a security against past returns on the market.
- The slope of the regression line (sometimes called the security's characteristic line) is defined as the beta coefficient for the security.

Comments on beta

- If beta = 1.0, the security is just as risky as the average stock.
- If beta > 1.0, the security is riskier than average.
- If beta < 1.0, the security is less risky than average.
- Most stocks have betas in the range of 0.5 to 1.5.

What is the market risk premium?

- Additional return over the risk-free rate needed to compensate investors for assuming an average amount of risk.
- Its size depends on the perceived risk of the stock market and investors' degree of risk aversion.
- Varies from year to year, but most estimates suggest that it ranges between 4% and 8% per year.

Calculating required rates of return

- k_{HT} = 8.0% + (15.0% - 8.0%)(1.30)
  = 8.0% + (7.0%)(1.30)
  = 8.0% + 9.1% = 17.10%
- k_M = 8.0% + (7.0%)(1.00) = 15.00%
- k_{USR} = 8.0% + (7.0%)(0.89) = 14.23%
- $k_{T\text{-}bill}$ = 8.0% + (7.0%)(0.00) = 8.00%
- k_{Coll} = 8.0% + (7.0%)(-0.87) = 1.91%

Expected vs. Required returns

	$\hat{k}$	k	
HT	17.4%	17.1%	Undervalued ($\hat{k} > k$)
Market	15.0	15.0	Fairly valued ($\hat{k} = k$)
USR	13.8	14.2	Overvalued ($\hat{k} < k$)
T-bills	8.0	8.0	Fairly valued ($\hat{k} = k$)
Coll.	1.7	1.9	Overvalued ($\hat{k} < k$)

An example: Equally-weighted two-stock portfolio

- Create a portfolio with 50% invested in HT and 50% invested in Collections.
- The beta of a portfolio is the weighted average of each of the stock's betas.

$$\beta_P = w_{HT}\,\beta_{HT} + w_{Coll}\,\beta_{Coll}$$
$$\beta_P = 0.5\,(1.30) + 0.5\,(-0.87)$$
$$\beta_P = 0.215$$

Calculating portfolio required returns

- The required return of a portfolio is the weighted average of each of the stock's required returns.

$$k_P = w_{HT}\,k_{HT} + w_{Coll}\,k_{Coll}$$
$$k_P = 0.5\,(17.1\%) + 0.5\,(1.9\%)$$
$$k_P = 9.5\%$$

- Or, using the portfolio's beta, CAPM can be used to solve for expected return.

$$k_P = k_{RF} + (k_M - k_{RF})\,\beta_P$$
$$k_P = 8.0\% + (15.0\% - 8.0\%)\,(0.215)$$
$$k_P = 9.5\%$$